General Instructions :

Maximum Marks : 80

1. *All questions are compulsory.*
2. *The question paper consists of 36 questions divided into 4 Sections A, B, C and D.*
3. *Section A comprises of 20 questions of 1 mark each, Section B comprises of 6 questions of 2 marks each, Section C comprises of 6 questions of 4 marks each and Section D comprises of 4 questions of 6 marks each.*
4. *There is no overall choice. However internal choice has been provided in 6 questions of 1 mark, 2 questions of 2 marks, 2 questions of 4 marks and 2 questions of 6 marks. You have to attempt only one of the alternatives in all such questions.*
5. *Write the serial number of questions before attempting.*
6. *Use of a calculator is not permitted.*

Section - A

Question numbers 1 to 10 carries 1 mark each. For each of these questions, four alternative choices have been provided of which only one is correct. Select the correct choice :

1. While shuffling a pack of 52 playing cards, 2 are accidentally dropped. Find the probability that the missing cards to be of different colours. [AI]

(A) $\frac{29}{52}$ **(B)** $\frac{1}{2}$

(C) $\frac{26}{51}$ **(D)** $\frac{27}{51}$

2. $\lim_{x\to 0}\frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$ is equal to :

(A) 2 **(B)** 0

(C) 1 **(D)** -1

OR

$\lim_{x\to 0}\frac{\operatorname{cosec} x-\cot x}{x}$ is equal to :

(A) $\frac{-1}{2}$ **(B)** 1

(C) $\frac{1}{2}$ **(D)** -1

3. *X*-axis is the intersection of two planes

(A) *XY* and *XZ* **(B)** *YZ* and *ZX*

(C) *XY* and *YZ* **(D)** None of these

4. If 9 times the 9^{th} term of an A.P. is equal to 13 times the 13^{th} term, then the 22^{nd} terms of the A.P. is

(A) 0 (B) 22
(C) 220 (D) 198

OR

If the sum of n terms of an A.P. is given by $S_n = 3n + 2n^2$, then the common difference of the A.P. is

(A) 3 (B) 2
(C) 6 (D) 4

5. Consider the first 10 positive integers. If we multiply each number by – 1 and, then add 1 to each number, the variance of the numbers, so obtain is AI

(A) 8.25 (B) 6.5
(C) 3.87 (D) 2.87

6. If x is a real number and $|x| < 3$, then

(A) $x \geq 3$ (B) $-3 < x < 3$
(C) $x \leq -3$ (D) $-3 \leq x < 3$

7. If the coefficients of 2nd, 3rd and the 4th terms in the expansion of $(1 + x)^n$ are in A.P., then value of n is

(A) 2 (B) 7
(C) 11 (D) 14

OR

If A and B are coefficient of x^n in the expansions of $(1 + x)^{2n}$ and $(1 + x)^{2n-1}$ respectively, then $\frac{A}{B}$ equals

(A) 1 (B) 2
(C) $\frac{1}{2}$ (D) $\frac{1}{n}$

[Hint : $\frac{A}{B} = \frac{{}^{2n}C_n}{{}^{2n-1}C_n} = 2$]

8. Equation of diagonals of the square formed by the lines $x = 0, y = 0, x = 1$ and $y = 1$ are

(A) $y = x, y + x = 1$ (B) $y = x, x + y = 2$
(C) $2y = x, y + x = \frac{1}{3}$ (D) $y = 2x, y + 2x = 1$

9. Equation of a circle which passes through (3, 6) and touches the axes is AI

(A) $x^2 + y^2 + 6x + 6y + 3 = 0$ (B) $x^2 + y^2 - 6x - 6y - 9 = 0$
(C) $x^2 + y^2 - 6x - 6y + 9 = 0$ (D) None of these

10. Following are the marks obtained by 9 students in a mathematics test 50, 69, 20, 33, 53, 39, 40, 65, 59. The mean deviation from the median is

(A) 9 (B) 10.5
(C) 12.67 (D) 14.76

Question numbers 11 to 15 carry 1 mark each. Write whether the statement is True or False.

11. The lines $ax + 2y + 1 = 0$, $bx + 3y + 1 = 0$ and $cx + 4y + 1 = 0$ are concurrent, if a, b and c are in GP.

12. Eighteen guests are to be seated, half on each side of a long. Four particular guests desire to sit on one particular side and three other on other side of the table. The number of ways in which the seating arrangements can be made is $\frac{11!}{5!6!}(9!)(9!)$

OR

There are 12 points in a plane of which 5 points are collinear, then the number of lines obtained by joining these points in pairs is ${}^{12}C_2 - {}^{5}C_2$.

13. The line $lx + my + n = 0$ will touch the parabola $y^2 = 4ax$, if $ln = am^2$

14. Let sets R and T be defined as

$R = \{x \in Z | \ x \text{ is divisible by } 2\}$

$T = \{x \in Z | \ x \text{ is divisible by } 6\}$. Then $T \subset R$

15. Let z_1 and z_2 be two complex number such that $|Z_1 + Z_2| = |Z_1| + |Z_2|$, then $|Z_1 - Z_2| = 0$.

OR

For any complex number z, the minimum value of $|z| + |z - 1|$ is 1.

Question numbers 16 to 20 carry 1 mark each.

16. Let f be the subset of $\mathbb{Z} \times \mathbb{Z}$ defined by

$f = \{(ab, a + b) : a, b \in \mathbb{Z}\}$. Is f a function from $\mathbb{Z}$ to $\mathbb{Z}$? Justify your answer.

17. Solve $5x < 24$, when $x \in N$

18. If $A = \{-1, 1\}$, find $A \times A \times A$.

19. What is the distance of the point (3, 4, 5) from the YZ plane ?

20. When a coin is tossed, write two events which are mutually exclusive and exhaustive.

OR

One number is chosen at random from the number 1 to 21. What is the probability that it is prime.

Section - B

Question numbers 21 to 26 carry 2 marks each.

21. 1 boy and 2 girls are in a room A and 3 boys and 1 girl are in room B. Write the sample space for the experiment in which room is selected and then a person.

22. Differentiate $\dfrac{x}{\sin x}$ with respect to x.

23. Solve $\dfrac{x+3}{x-1} > 0, x \in \mathbb{R}$.

24. Find the symmetric difference of sets $A = \{1, 3, 5, 6, 7\}$ and $B = \{3, 7, 8, 9\}$.

25. Evaluate the left hand and right hand limits of the following function at $x = 2$. Does $\lim_{x \to 2} f(x)$ exists ?

$f(x) = 2x + 3 \quad \text{if } x \le 2$

$f(x) = x + 5 \quad \text{if } x > 2$

OR

Show that $\lim_{x \to 4} \dfrac{|x-4|}{x-4}$ does not exist.

26. Prove that : $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \dfrac{x+y}{2}$

OR

Proved that : $\dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

Section - C

Question numbers 27 to 32 carry 4 marks each.

27. Prove that :

$$\cos A \cos 2A \cos 4A \cos 8A = \frac{\sin 16A}{16 \sin A}$$

OR

If $\tan A - \tan B = x$, $\cot B - \cot A = y$, prove that $\cot (A - B) = \dfrac{1}{x} + \dfrac{1}{y}$

28. Show that the middle term in the expansion of $(1+x)^{2n}$ is $\dfrac{\{1.3.5.7...(2n-1)\}2^n x^n}{n!}$ where $n \in N$.

OR

If the first three terms in the expansion of $(a+b)^n$ are 27, 54 and 36 respectively, then find a, b and n.

29. Find the co-ordinates of the foci, the vertices, the eccentricity and the length of the latus rectum of hyperbola $9y^2 - 4x^2 = 36$.

30. In how many ways 7 positive and 5 negative signs can be arranged in a row so that no two negative signs occur together?

31. Find real valued θ such that $\dfrac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.

32. Find the mean and variance for first n natural numbers.

Section - D

Question numbers 33 to 36 carry 6 marks each.

33. Using the properties of sets and their complements prove that

$$(A \cup B) - C = (A - C) \cup (B - C)$$

34. If the A.M. between r^{th} and s^{th} terms of an A.P. be equal to A.M. between r^{th} and s^{th} terms of the A.P., then show that $p + q = r + s$.

OR

Find the value of the expression :

$$\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$$

35. Solve the following system of linear inequalities

$$3x + 2y \geq 24,\ 3x + y \leq 15,\ x \geq 4.$$

36. Let S be the sum, P be the product and R be the sum of reciprocals of n terms in a G.P.. Prove that $P^2R^n = S^n$.

OR

If $f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & \text{when } x \neq \dfrac{\pi}{2} \\ 3, & \text{when } x = \dfrac{\pi}{2} \end{cases}$ and $\lim_{x \to \pi/2} f(x) = f\left(\dfrac{\pi}{2}\right)$, then find the value of k.